PREFACE

This issue of *Logical Methods in Computer Science* is composed of selected papers submitted by participants of the Workshop "Continuity, Computability, Constructivity: From Logic to Algorithms", held in the Georg-Vollmar-Akademie, Schloss Aspenstein, a conference centre located in the magnificent surroundings of the Bavarian prealps, in mid September 2015.

The workshop was partially funded by the German Science Association (DFG), the Japan Society for the Promotion of Science (JSPS), as well as the European Union. It was the fifth in a series of workshops aimed at bringing together researchers from exact real number computation, computable analysis, effective descriptive set theory, constructive analysis, and related fields. The overall objective is to apply logical methods in these disciplines to provide a sound foundation for obtaining exact and correct algorithms. At the same time, the conference was the final annual meeting of the COMPUTAL project, a research network between Europe, Russia, South Africa, and Japan funded by the European Union under the FP7-IRSES programme scheme.

A central aim for the project collaboration has been laying the grounds for the generation of efficient and verified software in engineering applications. A pivotal problem in current scientific computations is still the dissociation between the mathematical theory and its implementation in computer programs: Implementations of the reals use floating-point numbers of fixed finite precision, whereas proofs of program properties are given on the basis of the classical theory of the real number field.

Ways out of this problem are promoted under the slogan *Exact Real Arithmetic* in which real numbers are taken as first-class citizens. While any computation can only exploit a finite portion of its input in finite time, increased precision is always available by continuing the computation process.

Well developed practical and theoretical bases for exact real number computation and, more generally, computable analysis are provided by Scott's Domain Theory and Weihrauch's Type-Two Theory of Effectivity. In both theories real numbers and similar ideal objects are represented by infinite streams of finite objects. They can locally be manipulated by Turing machines. In contrast to the theory of computing with finite data, the study of computing with infinite data crucially depends on topological and measure-theoretic considerations.

A related approach is pursued in constructive analysis by using intuitionistic logic. As is well known, proofs of existential statements in this logic allow the extraction of algorithms computing the object that is stated to exist. By their nature these algorithms are correct.

Many important problems are either inherently difficult to compute, or even not computable at all. To measure their computational hardness or the degree on non-computability one uses hierarchies, or compares them with certain well-known master problems. In many

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applications operators on function spaces are of central relevance. So, not only the computational complexity of functions on the reals, but also of such operators needs be investigated.

The present issue contains contributions to all these areas.

As usual, there are many people to be thanked. This is in particular true for the organising committee of the Aspenstein meeting. They did a wonderful job. Moreover, we want to thank the referees for having taken the burden of carefully reading and commenting the submissions. Last, but not least, we are very grateful to the Editor-in-Chief of *Logical Methods in Computer Science* for the opportunity to publish in this issue of the journal.

The papers have undergone a rigorous reviewing process in accordance with the standards set by *Logical Methods in Computer Science*.

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